# Transition Moments between $w^{\mathbf{3}} \boldsymbol{\Pi}_{\mathrm{g}}$ State and the First Three ${ }^{3} \boldsymbol{\Sigma}_{u}$ and ${ }^{3} \boldsymbol{\Pi}_{u}$ States of the Hydrogen Molecule ${ }^{\dagger}$ 

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#### Abstract

An explicitly correlated variational electronic wave function is obtained for the $w^{3} \Pi_{\mathrm{g}}$ state of the hydrogen molecule and the Born-Oppenheimer potential energy curve as well as adiabatic corrections are given for internuclear distances $1 \leq R \leq 50$ au. The electronic dipole transition moments between $w^{3} \Pi_{\mathrm{g}}$ and $b^{3} \Sigma_{\mathrm{u}}$, $e^{3} \Sigma_{\mathrm{u}}, f^{3} \Sigma_{\mathrm{u}}, c^{3} \Pi_{\mathrm{u}}, d^{3} \Pi_{\mathrm{u}}$, and $k^{3} \Pi_{\mathrm{u}}$ states for $0.6 \leq R \leq 20$ au are computed. These are the first accurate ab initio results for the above transitions. Also, the adiabatic vibrational levels are presented for $N=1$.


## Introduction

Recently, due to new sophisticated techniques the experimental investigation of excited states of the hydrogen molecule has shown considerable progress allowing the determination of energies of the rovibrational states with very high accuracy, often exceeding $0.05 \mathrm{~cm}^{-1}$ (see e.g. refs $1-13$ and references therein). Hence, accurate theoretical description of the hydrogen molecule becomes more important and further systematic ab initio investigations are required. The determination of the electronic wave functions of the molecule and the Born-Oppenheimer (BO) energies is often necessary parts of such calculations (see e.g. refs $3,14-20$ ). As is well-known, for diatomic two-electron molecules the variational calculations in the confocal elliptical coordinates with explicitly correlated electrons allows one to achieve high accuracy (see e.g. refs 21-29). The data obtained in this way are also used to verify the accuracy of new methods developed for the description of larger molecular systems (see e.g. refs 30-33).

Recently, new accurate ab initio results for a number of triplet states have been published. ${ }^{28,29}$ One of these papers ${ }^{28}$ presents the Born-Oppenheimer potential energy curves and adiabatic corrections for three lowest ${ }^{3} \Pi_{\mathrm{u}}$ states $(c, d, k)$ and three lowest ${ }^{3} \Pi_{\mathrm{g}}$ states $(i, r, w)$; some of these data had been available before publication. In the second paper, ${ }^{29}$ three lowest ${ }^{3} \Sigma_{\mathrm{g}}$ states ( $a, h$, $g)$, three lowest ${ }^{3} \Sigma_{\mathrm{u}}$ states $(b, e, f)$, three lowest ${ }^{3} \Pi_{\mathrm{u}}$ states $(c, d$, $k$ ), and two lowest ${ }^{3} \Pi_{\mathrm{g}}$ states $(i, r)$ were computed, or recomputed, and used to determine the potential energy curves, adiabatic corrections, and electronic dipole transition moments between these states with uniform accuracy in a wide range of internuclear distances.

The present work was undertaken in order to complete the calculations of the dipole transition moments by adding transitions from the third state of ${ }^{3} \Pi_{\mathrm{g}}$ symmetry, and these results seem to be the first accurate ab initio calculations of the transitions moments involving $w^{3} \Pi_{g}$ state. The computed transitions will be used in future in nonadiabatic calculations.

The method of computation of the wave function is briefly recalled in the following section. There the details of accurate calculations performed for the $w$ state are given. In the following sections the results of the calculations of the Born-Oppenheimer

[^0]potential, adiabatic corrections, and adiabatic vibrational energies for $N=1$ are presented. The last section reports results of calculations of the electronic dipole transition moments between $w^{3} \Pi_{\mathrm{g}}$ and $b^{3} \Sigma_{\mathrm{u}}, e^{3} \Sigma_{\mathrm{u}}, f^{3} \Sigma_{\mathrm{u}}, c^{3} \Pi_{\mathrm{u}}, d^{3} \Pi_{\mathrm{u}}$, and $k^{3} \Pi_{\mathrm{u}}$ states.

Atomic units are used throughout unless otherwise stated; the energy conversion factor to wavenumber is 1 hartree $=$ $219474.631 \mathrm{~cm}^{-1}$, the reduced mass of the nuclei is $\mu=$ 918.0764.

## The Wave Function

In this work the generalized James-Coolidge wave function developed in refs 19,23 , and $34-36$ is used to describe the electronic motion of the hydrogen molecule. It has a form of an expansion in elliptic coordinates of the two electrons

$$
\begin{equation*}
\psi=\sum_{k} c_{k} G_{k}(1,2) \tag{1}
\end{equation*}
$$

where

$$
\begin{equation*}
G_{k}(1,2)=\left(x_{1}+i y_{1}\right)^{\Lambda} g_{k}(1,2) \pm\left(x_{2}+i y_{2}\right)^{\Lambda} g_{k}(2,1) \tag{2}
\end{equation*}
$$

and

$$
\begin{array}{r}
g_{k}(1,2)=\exp \left(-\alpha \xi_{1}-\bar{\alpha} \xi_{2}\right) \varsigma^{v_{k}} \xi_{1}^{r_{k}} \eta_{1}{ }^{s_{k}} \xi_{2}^{\bar{r}_{k}} \eta_{2}{ }^{\bar{s}_{\mathrm{k}}}\left\{\operatorname { e x p } \left(\beta \eta_{1}+\right.\right. \\
\left.\left.\bar{\beta} \eta_{2}\right)+(-1)^{s_{k}+\bar{s}_{k}+\Lambda+p} \exp \left(-\beta \eta_{1}-\bar{\beta} \eta_{2}\right)\right\} \tag{3}
\end{array}
$$

Assuming that the internuclear axis coincides with the $z$ axis, in this expansion, $\xi_{j}$ and $\eta_{j}$ are elliptic coordinates of the two electrons, $x_{j}$ and $y_{j}$ their Cartesian coordinates, and $\varsigma=2 r_{12} / R$, where $r_{12}$ and $R$ denote the interelectronic and internuclear distances respectively; $\Lambda$ denotes the angular momentum projection quantum number (equal to 0 for $\Sigma$ states and 1 for $\Pi$ states); $p=0,1$ for g and u symmetry respectively; $\pm$ in eq 2 refers to singlet and triplet states, respectively; $c_{k}, \alpha, \bar{\alpha}, \beta, \bar{\beta}$ are variational parameters and the basis set is defined by the set of exponents $v_{k}, r_{k}, s_{k}, \bar{r}_{k}, \bar{s}_{k}$.

For the $w$ state, the terms in the expansions (1)-(3) were chosen from the set of exponents fulfilling the conditions: $v \leq$ 5 and $v+r+s \leq 8$. The selection of terms was performed in the usual way. ${ }^{26}$ First, for a short expansion, the nonlinear parameters were optimized for several values of $R(1,2,5,10$, 15). Next, from the above set of exponents the individual terms were tested-terms which gave the biggest decrease of the
energy were added to the expansion and nonlinear parameters were reoptimized. This procedure was repeated several times. When 300 terms were selected, the nonlinear parameters were optimized for 91 values of $R$ from the interval between 1 and 50 au and next the selection of terms was continued. As a result, a 478 -term wave function was generated. The phase of this function is chosen in agreement with the convention adopted earlier (see e.g. refs 25 and 29).

## Born-Oppenheimer Energies and Adiabatic Corrections

The total nonrelativistic Hamiltonian of the molecule in the center of mass system is

$$
\begin{equation*}
H=H_{0}+H^{\prime} \tag{4}
\end{equation*}
$$

where $H_{0}$ is the clamped nuclei Hamiltonian and $H^{\prime}$ is the operator depending on the mass of the nuclei. For homonuclear molecules $H^{\prime}$ can be expressed as a sum of two parts

$$
\begin{equation*}
H^{\prime}=H_{1}^{\prime}+H_{2}^{\prime} \tag{5}
\end{equation*}
$$

where

$$
\begin{gather*}
H_{1}^{\prime}=-\frac{1}{2 \mu} \Delta_{\mathrm{R}}  \tag{6}\\
H_{2}^{\prime}=-\frac{1}{2 \mu}\left(\nabla_{1}+\nabla_{2}\right)^{2} \tag{7}
\end{gather*}
$$

and $\mu$ is the reduced mass of the nuclei.
Let the electronic wave function, $\psi$, be the approximate solution of the clamped nuclei Schrödinger equation. Then, in the adiabatic approximation, the total molecular wave function, $\Phi$ has the form of a product of a rotational function, $\psi^{\text {rot }}$, adiabatic vibrational function, $\chi(R)$, and the electronic wave function, $\psi$ :

$$
\begin{equation*}
\Phi=\frac{1}{R} \chi(R) \psi^{\mathrm{rot}} \psi \tag{8}
\end{equation*}
$$

The use of this product in the Schrödinger equation with the total Hamiltonian (4) leads to the well-known one-dimensional equation for nuclear motion

$$
\begin{equation*}
\left\{-\frac{1}{2 \mu} \frac{\mathrm{~d}^{2}}{\mathrm{~d} R^{2}}+U(R)-E_{\mathrm{v}, N}\right\} \chi_{\mathrm{v}, N}=0 \tag{9}
\end{equation*}
$$

where

$$
\begin{equation*}
U(R)=E^{\mathrm{BO}}(R)+\left\langle\psi \psi_{N}^{\mathrm{rot}}\right| H^{\prime}\left|\psi \psi_{N}^{\mathrm{rot}}\right\rangle \tag{10}
\end{equation*}
$$

$E^{\mathrm{BO}}(R)$ in eq 10 is the Born-Oppenheimer energy and the second term consists of the centrifugal potential as well as the adiabatic corrections. According to the convention adopted in ref 23, this splitting is written as

$$
\begin{equation*}
\left\langle\psi \psi_{N}^{\mathrm{rot}}\right| H^{\prime}\left|\psi \psi_{N}^{\mathrm{rot}}\right\rangle=E^{\prime}(R)+\frac{N(N+1)}{2 \mu R^{2}} \tag{11}
\end{equation*}
$$

and all N -independent parts of the expectation value of $H^{\prime}$ are included in the adiabatic correction $E^{\prime}(R)$. The method of calculation of the adiabatic correction is given in, e.g., ref 23. Part of this correction connected with the operator $\mathrm{H}_{2}^{\prime}$ resolves itself into the form of the expectation value with electronic wave function only. For the states with $\Lambda \neq 0$, in the second part,

TABLE 1: Born-Oppenheimer Energies and Adiabatic Corrections for the $\boldsymbol{w}$ State of $\mathbf{H}_{\mathbf{2}}$

| $R$ | $E^{\mathrm{BO}}$ | $\mathrm{d} E^{\mathrm{BO}} / \mathrm{d} R$ | $D\left(\mathrm{~cm}^{-1}\right)$ | $E\left(\mathrm{~cm}^{-1}\right)$ |
| ---: | ---: | ---: | ---: | ---: |
| 1.00 | -0.4718824472 | -0.521102919 | -18364.125 | 559.328 |
| 1.20 | -0.5491018286 | -0.277338725 | -1416.429 | 406.426 |
| 1.40 | -0.5901480429 | -0.145837911 | 7592.173 | 312.915 |
| 1.50 | -0.6025087511 | -0.103315690 | 10305.035 | 279.202 |
| 1.60 | -0.6111456086 | -0.070855614 | 12200.606 | 251.454 |
| 1.70 | -0.6169292695 | -0.045895767 | 13469.973 | 228.342 |
| 1.80 | -0.6205131579 | -0.026603166 | 14256.546 | 208.894 |
| 1.90 | -0.6223940601 | -0.011644480 | 14669.356 | 192.387 |
| 2.00 | -0.6229534169 | -0.000033729 | 14792.121 | 178.269 |
| 2.10 | -0.6224877873 | 0.008965328 | 14689.927 | 166.121 |
| 2.20 | -0.6212287991 | 0.015912327 | 14413.611 | 155.612 |
| 2.50 | -0.6143431646 | 0.028244717 | 12902.389 | 131.620 |
| 3.00 | -0.5984142200 | 0.033327900 | 9406.390 | 109.705 |
| 3.50 | -0.5824466611 | 0.029418572 | 5901.916 | 127.899 |
| 3.80 | -0.5744935777 | 0.022829342 | 4156.415 | 281.871 |
| 4.00 | -0.5705534054 | 0.016772735 | 3291.648 | 426.670 |
| 4.20 | -0.5674829783 | 0.014833988 | 2617.767 | 299.256 |
| 4.50 | -0.5628672245 | 0.015776261 | 1604.726 | 138.467 |
| 5.00 | -0.5555229895 | 0.012734163 | -7.147 | 104.372 |
| 5.50 | -0.5509076448 | 0.004885277 | -1020.099 | 181.463 |
| 5.59 | -0.5505492301 | 0.003086772 | -1098.761 | 187.614 |
| 6.00 | -0.5505902159 | -0.002267901 | -1089.766 | 112.010 |
| 6.50 | -0.5520226387 | -0.002922574 | -775.386 | 74.451 |
| 7.00 | -0.5533069342 | -0.002171560 | -493.515 | 69.123 |
| 7.50 | -0.5542013644 | -0.001435398 | -297.211 | 68.167 |
| 7.80 | -0.5545779498 | -0.001087363 | -214.560 | 68.103 |
| 8.00 | -0.5547756460 | -0.000894600 | -171.170 | 68.144 |
| 8.50 | -0.5551256069 | -0.000531184 | -94.363 | 68.321 |
| 9.00 | -0.5553289415 | -0.000300695 | -49.736 | 68.375 |
| 9.50 | -0.5554415701 | -0.000162189 | -25.017 | 68.222 |
| 10.00 | -0.5555009777 | -0.000083147 | -11.979 | 67.951 |
| 10.50 | -0.5555306644 | -0.000040113 | -5.463 | 67.678 |
| 11.00 | -0.5555444825 | -0.000017673 | -2.430 | 67.459 |
| 11.50 | -0.5555501902 | -0.000006505 | -1.178 | 67.303 |
| 12.00 | -0.5555519638 | -0.000001287 | -0.788 | 67.193 |
| 12.50 | -0.5555519756 | 0.000000892 | -0.786 | 67.115 |
| 13.00 | -0.5555513152 | 0.000001583 | -0.931 | 67.056 |
| 14.00 | -0.5555497682 | 0.000001324 | -1.270 | 66.969 |
| 15.00 | -0.5555487876 | 0.000000642 | -1.485 | 66.901 |
| 16.00 | -0.5555484245 | 0.000000120 | -1.565 | 66.844 |
| 17.00 | -0.5555484767 | -0.000000195 | -1.554 | 66.795 |
| 18.00 | -0.5555487631 | -0.000000359 | -1.491 | 66.754 |
| 20.00 | -0.5555496055 | -0.000000448 | -1.306 | 66.690 |
| 25.00 | -0.5555515897 | -0.000000327 | -0.870 | 66.575 |
| 30.00 | -0.5555529103 | -0.000000209 | -0.581 | 66.479 |
| 35.00 | -0.5555537517 | -0.000000133 | -0.396 | 66.435 |
| 40.00 | -0.5555542898 | -0.000000086 | -0.278 | 66.418 |
| 45.00 | -0.5555546409 | -0.000000057 | -0.201 | 66.412 |
| 50.00 | -0.5555548765 | -0.000000039 | -0.149 | 66.409 |
|  | 0 |  |  |  |

coming from the operator $H_{1}^{\prime}$, the expectation value must be evaluated over electronic and rotational coordinates.

The Born-Oppenheimer energies and the adiabatic corrections were computed for 361 internuclear distances. A complete set of results can be obtained through www.phys.uni.torun.pl/ ftp/publications/ifiz/luwo/h2trans.00. Some of them are presented in Table 1, where besides the BO energies, $E^{\mathrm{BO}}(R)$, and adiabatic corrections, $E^{\prime}$, (in $\mathrm{cm}^{-1}$ ) the derivatives $\mathrm{d} E^{\mathrm{BO}} / \mathrm{d} R$, obtained from the virial theorem, and the dissociation energies, $D$ (in $\mathrm{cm}^{-1}$ ), are also given. The accuracy of these calculations was tested by recomputing some points with real*16 arithmetic. The energy curve obtained in this work is generally slightly lower than that computed by Kołos and Rychlewski. ${ }^{28}$

The results are also displayed in Figures 1 and 2. It is seen in Figure 1 that besides the typical minimum at $R=2.0$ the BO potential curve exhibits a very shallow second minimum at $R=12.2$, located above the dissociation limit.

The total adiabatic corrections are plotted in Figure 2. One can observe the large corrections at about $R=4.0$ and $R=$


Figure 1. Born-Oppenheimer electronic energy curve of the $w^{3} \Pi_{g}$ state (in au).


Figure 2. Total adiabatic corrections of the $w^{3} \Pi_{\mathrm{g}}$ state $\left(\mathrm{in} \mathrm{cm}^{-1}\right)$.
TABLE 2: Vibrational Energies, Dissociation Energies, Vibrational Quanta, and Rotational Constants for Vibrational Levels in the $\boldsymbol{w}$ State of $\mathbf{H}_{2}$

| $v$ | $E_{\mathrm{v}}$ | $D_{\mathrm{v}}$ | $\Delta G(v+1 / 2)$ | $\left\langle R_{2}\right\rangle$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | -0.61668703 | 13483.178 | 2167.372 | 0.2411 |
| 1 | -0.60681176 | 11315.806 | 2036.659 | 0.2281 |
| 2 | -0.59753206 | 9279.147 | 1908.055 | 0.2155 |
| 3 | -0.58883832 | 7371.092 | 1779.916 | 0.2030 |
| 4 | -0.5072843 | 5591.176 | 1646.717 | 0.1905 |
| 5 | -0.57322543 | 3944.459 | 1465.758 | 0.1770 |
| 6 | -0.56654694 | 2478.701 | 1170.484 | 0.1561 |
| 7 | -0.56121383 | 1308.216 | 1055.018 | 0.1315 |
| 8 | -0.55640681 | 253.198 |  | 0.1288 |

5.59 and a very small maximum at $R=8.9$. Large corrections indicate changes in the character of the molecular states due to avoided crossings and strong nonadiabatic interactions. ${ }^{21-23,29}$ As was pointed out in ref 28 , the maximum at $R=4.0$ is due to an avoided crossing with a repulsive diabatic curve and the maximum at $R=5.59$ arises from interaction with a higher state. The third small maximum at $R=8.9$ can indicate an avoided crossing with the $r^{3} \Pi_{\mathrm{g}}$ state for which the adiabatic corrections have a maximum in the same region (at $R=7.8$ ).

## Adiabatic Vibrational Levels

The adiabatic potential (10) obtained in this work was used in the equation for nuclear motion (9) to calculate the rovibrational energies. The results for $N=1$ are listed in Table 2. In the first column the vibrational energies, $E_{\mathrm{v}}$, are given, next, the dissociation energies, $D_{\mathrm{v}}$, further, vibrational quanta denoted as $\Delta G$ and in the last column the expectation values of $R^{-2}$. The latter are proportional to the rotational constants: $B_{\mathrm{v}}=$ $\langle v| R^{-2}|v\rangle / 2 \mu$. All vibrational states are localized within inner

TABLE 3: Dipole Transition Moments Involving $\boldsymbol{w}$ State of $\mathbf{H}_{2}$

| $R$ | $w \rightarrow b$ | $w \rightarrow e$ | $w \rightarrow f$ | $w \rightarrow c$ | $w \rightarrow d$ | $w \rightarrow k$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0.60 | -0.4155 | -1.0498 | 2.9981 | -0.4292 | -1.1773 | 3.8117 |
| 0.80 | -0.3956 | -0.9206 | 2.3540 | -0.4280 | -1.1766 | 3.7998 |
| 1.00 | -0.3648 | -0.7635 | 1.6774 | -0.4269 | -1.1814 | 3.8287 |
| 1.25 | -0.3160 | -0.5663 | 0.9446 | -0.4254 | -1.2138 | 4.0733 |
| 1.75 | -0.2320 | -0.3058 | 0.1179 | -0.4190 | -1.2344 | 4.2649 |
| 2.00 | -0.2084 | -0.2467 | -0.0530 | -0.4121 | -1.2622 | 4.5103 |
| 2.25 | -0.1957 | -0.2160 | -0.1295 | -0.4009 | -1.2895 | 4.8064 |
| 2.50 | -0.1917 | -0.2084 | -0.1423 | -0.3828 | -1.3116 | 5.1549 |
| 2.75 | -0.1957 | -0.2185 | -0.1038 | -0.3535 | -1.3270 | 5.6043 |
| 3.00 | -0.2082 | -0.2450 | -0.0145 | -0.3052 | -1.3314 | 1.7941 |
| 3.25 | -0.2321 | -0.2918 | 0.1452 | -0.2215 | -1.3140 | 1.7038 |
| 3.50 | -0.2743 | -0.3715 | 0.4426 | -0.0622 | -1.2417 | 1.9485 |
| 3.75 | -0.3434 | -0.5100 | 1.0795 | 0.2784 | -0.9850 | 2.5428 |
| 4.00 | -0.3887 | -0.6687 | 2.3751 | 0.9016 | -0.1461 | 3.7549 |
| 4.50 | -0.1434 | -0.4601 | 3.9015 | 1.4049 | 2.1697 | 5.8097 |
| 5.00 | 0.0483 | -0.1288 | -0.0370 | 1.3225 | 2.9228 | 5.8241 |
| 5.50 | 0.2353 | 0.3730 | -1.4924 | 1.1851 | 2.4656 | 4.9467 |
| 6.00 | 0.3789 | 0.9358 | -1.6405 | 0.8175 | 0.9554 | 4.4064 |
| 6.50 | 0.4161 | 1.1993 | -1.4138 | 0.5115 | -0.0766 | 4.2455 |
| 7.00 | 0.4253 | 1.3429 | -1.1985 | 0.3014 | -0.9059 | 4.1460 |
| 7.50 | 0.4275 | 1.4496 | -1.0140 | 0.1279 | -1.8132 | 3.8889 |
| 8.00 | 0.4267 | 1.5432 | -0.8389 | -0.0347 | -2.5408 | 3.5181 |
| 8.50 | 0.4236 | 1.6280 | -0.6587 | -0.1936 | -2.8999 | 3.2814 |
| 9.00 | 0.4187 | 1.7005 | -0.4683 | -0.3456 | -3.0124 | 3.2190 |
| 9.50 | 0.4126 | 1.5550 | -0.2698 | -0.4838 | -3.0000 | 3.2601 |
| 10.00 | 0.4059 | 1.7874 | -0.0682 | -0.6025 | -2.9298 | 3.3458 |
| 10.50 | 0.3993 | 1.7967 | 0.1307 | -0.7002 | -2.8350 | 3.4433 |
| 11.00 | 0.3934 | 1.7842 | 0.3203 | -0.7784 | -2.7329 | 3.5382 |
| 11.50 | 0.3882 | 1.7544 | 0.4931 | -0.8401 | -2.6310 | 3.6248 |
| 12.00 | 0.3838 | 1.7140 | 0.6422 | -0.8886 | -2.5325 | 3.7024 |
| 13.00 | 0.3771 | 1.6292 | 0.8602 | -0.9568 | -2.3478 | 3.8331 |
| 15.00 | 0.3700 | 1.5302 | 1.0485 | -1.0234 | -2.0172 | 4.0269 |
| 20.00 | 0.3741 | 1.5491 | 1.0108 | -0.9829 | -1.0428 | 4.3854 |








Figure 3. Transition moments involving the $w^{3} \Pi_{\mathrm{g}}$ state (in au).
potential well, the outer well is too shallow to support a bound state. The presented vibrational energies are calculated in the adiabatic approximation, in order to compare them with experimental data (if available) the nonadiabatic calculations are necessary.

## Transition Moments

Dipole transition moments have been computed in several recent studies. ${ }^{5,16,21,25,29}$ It was shown in ref 25 that the dipole formula is more suitable for numerical calculations than the velocity formula. So, here, as in a previous paper, ${ }^{29}$ the transition moments, $\mathbf{M}$, between states $\psi_{1}$ and $\psi_{2}$ were computed from the dipole formula

$$
\begin{equation*}
\mathbf{M}=\left\langle\psi_{1}\right| \mathbf{r}_{1}+\mathbf{r}_{2}\left|\psi_{2}\right\rangle \tag{12}
\end{equation*}
$$

For $\Sigma \rightarrow \Pi$ transitions, the wave function of the $\Pi$ state is inserted on the left side of eq 12 so the nonvanishing components of the transition moments are $M_{+}=\left(M_{x}+i M_{y}\right) /$ $\sqrt{2}$ for the $\Sigma \rightarrow \Pi$ transitions and $M_{z}$ for the $\Pi \rightarrow \Pi$ transitions. All transitions are computed for internuclear distances from the interval [0.6-20]. Some of the results are given in Table 3 and in Figure 3. Full tables are available under file name www.phys.uni.torun.pl/ftp/publications/ifiz/luwo/h2trans. 00 .

The transition moments presented in this work have not been investigated before; together with those considered in ref 29 , they correspond to all possible transitions between three lowest states of ${ }^{3} \Sigma_{\mathrm{g}},{ }^{3} \Sigma_{\mathrm{u}},{ }^{3} \Pi_{\mathrm{g}}$, and ${ }^{3} \Pi_{\mathrm{u}}$ symmetries. In all those cases, one can observe an irregular behavior of the moments, in particular, in the regions where the adiabatic corrections of the involved states are large. Experience with previously investigated transitions (where the comparison with experimental data was possible) shows that nonadiabatic interactions are important. The investigation of these effects is in preparation in this laboratory.

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